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KATHOLIEKE HOGESCHOOL TILBURG

REEKS TER DISCUSSIE

A NEW CONCEPT FOR ALLOCATION OF JOINT COSTS:
Stepwise reduction of costs proportional to joint savings

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FACULTEIT DER ECONOMISCHE WETENSCHAPPEN

NOTE ON ALLOCATION OF JOINT COSTS: Stepwise reduction of costs proportional to joint savings

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1. Introduction

Recently Ståhl analysed the results of a game on cost allocation in water resources: see Ståhl (1982). He introduced seven solution concepts for obtaining a unique allocation of the total costs of a coalition. He evaluated these seven concepts using 16 actual game results. I am interested in his study because of a similar problem: how to allocate the fixed costs of an information system or computing centre over the participating departments? Although the solution to such type of problems is finally a political one, obtained via negotiations, we should try to find generally accepted allocation principles to support the political allocation process. Kleynen and Van Reeken (1982) proposed one concept that was not among those discussed by Ståhl. This note compares Ståhl's solution principles to the latter concept.

2. Five principles

There are n parties interested in forming coalitions among each other for some activity in order to obtain a cost reduction as compared to the costs of doing the activity on its own. For a grand coalition to be formed of all N parties (and for each coalition of m < N) certain principles may apply:

1. The Full Cost principle: payments made by parties total the costs of the coalition:

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$$\sum_{i=1}^{n} x_i = c(N),$$

where x_i = payment made by party i

and c(N) = costs of grand coalition of all n parties.

2. The Individual Rationality principle: payments made by party i are not higher than its going alone costs, c(i):

$$x_i \leq c(i) \text{ for all } i.$$

3. The Group Rationality principle: payments made by parties of every coalition which is smaller than the grand coalition are not higher than the costs of that coalition on its own:

$$\sum_{i \in S} x_i \leq c(S) \text{ for all } S \subset \{1,\ldots,n\}$$

If allocation of costs satisfies these three principles the solution belongs to "the core". There may be more solutions in the core.
Two additional principles can be formulated:

4. The Monotonicity principle: if the costs of the coalition go down, no one should be charged more and if total costs go up, no one shall pay less:

$$c(N) \lessgtr c'(N) \Rightarrow x_i \lessgtr x_i' \text{ for all } i.$$

5. The Causality principle: if a party never contributes to any cost savings when joining with other parties or coalitions, this party should not realize any cost savings above his go alone costs:

$$c(S) + c(i) = c(S+i) \text{ for all } S \subset \{1,\ldots,n\} \Rightarrow x_i = c(i).$$

3. The concept

The concept "Stepwise reduction of costs proportional to joint savings" implies a step by step formation of coalitions between two parties: first between single parties, then among two-party coalitions and remaining singles and so up to a grand coalition. An example hereafter will clarify this procedure.
Unlike the Shapley Value (see Shapley (1953)) the former concept fixes the order in which coalitions can take place.
The order is fixed by the following two principles:

6. The reduction of costs will be proportional to joint savings. (equity principle).
   For illustration purposes suppose that the individual costs of parties A and B are 4 respectively 6 and that the two-party coalition AB costs 8. Then the joint savings are 20% and so both A and B will obtain a cost reduction of 20%.

7) Each party tries to realize the largest reduction for itself. (economic principles).
   Suppose A has the opportunity to join with B (see above) but also with C (and that the latter coalition will result in a 30% cost reduction; then A will prefer the coalition with C to the one with B.*).

The stepwise procedure will be explained as follows. Each party first identifies possible partners, i.e. partners with which a two-party coalition will lead to lower total costs than when each of the two parties goes alone. Each party then tries to form the two-party coalition with the largest relative cost reduction. This coalition is formed indeed, provided there is a two-party coalition for which this holds for

*) The problem of intransivity (A prefers C, C prefers B and B prefers A) can not occur, since when

$$\frac{c(AC)}{c(A)+c(B)} < \frac{c(AB)}{c(A)+c(B)} \text{ and } \frac{c(BC)}{c(B)+c(C)} < \frac{c(AC)}{c(A)+c(C)} \text{ then } \frac{c(BC)}{c(B)+c(C)} < \frac{c(AB)}{c(A)+c(B)}$$

and B will prefer C.

both parties. When such a coalition has been formed, the remaining parties repeat this process until each party has find its partner or remains single due to lack of profitable partners. So we see a bilateral coalition formation.

N.B. In the example given by StÅhl, there are six possible two-party coalitions:

AH with a reduction of total costs to	88,88%
HK	82,03%
HL	75,85%
KM	99,15%
LM	84,76%
MT	92,10%

So, A, K and L want to join with H; M wants to join with L: T wants to join with M; and H wants to join with L.
The only two-party coalition for which there is a preference on both sides, is HL. This coalition is formed. So A will remain single; K and T will try M; and M will try T. The coalition MT will be formed, and thus also K will remain single.

Then a new round starts in which the two-party coalitions must be seen as parties. The process described above is repeated among these parties. The result of the second round can be either a four-party coalition when two two-party coalitions join, or a three-party coalition when a two-party coalition joins with one that remained single in the first round. Now there are two ways of calculating the reduction in total costs for such a three-party coalition (and likewise for the four-party coalition):
a) against the sum of the individual costs
b) against the sum of the two-party coalition costs and the individual cost of the third party.

The rationality principle implies that the two-party coalition will only accept a three-party coalition when their payment in the three-party coalition is smaller than in the two-party coalition. Since it can be proved*) that for the two-party coalition the reduction sub a is always smaller than the cumulative reduction sub b, the two-party coalition will prefer b to calculate the reduction.

N.B. In the example the possibilities are:

A(HL) with a reduction of total costs to	92,06%
HKL ≡ K(HL)	75,91%

since coalitions of MT and A or K; or of HL and MT do not pay. So HL and K will prefer each other and form a coalition. In the third round HKL is able to reduce their costs even further by joining with A (reduction to 99,47%) since a coalition with MT will pay less.
Finally in the fourth round the grand coalition will be formed with a further reduction of costs to 94,86%.

N.B. Percentages are always relative to costs in the preceeding step; see b above.

The final cost distribution will be as follows:

A:	21,95			x 0,9947	x 0,9486 =	20,71
H:	17,08	x 0,7585	x 0,7591	x 0,9947	x 0,9486 =	9,28
K:	10,91		x 0,7591	x 0,9947	x 0,9486 =	7,81
L:	15,88	x 0,7585	x 0,7591	x 0,9947	x 0,9486 =	8,63
M:	20,81	x 0,9210			x 0,9486 =	18,18
T:	21,98	x 0,9210			x 0,9486 =	19,20
						83,81

In formula:

$$x_i = c(i) \cdot \prod_{S' \vee S'' = S \subset V} \left\{\frac{c(S)}{c(S') + c(S'')}\right\};$$

*) When $c(AB) = \alpha\,(c(A)+c(B))$, $0 < \alpha < 1$, then

$$\frac{c(ABC)}{c(A)+c(B)+c(C)} > \frac{c(ABC)}{c(AB)+c(C)} \cdot \frac{c(AB)}{c(A)+c(B)} \quad \text{since } c(C) > \alpha . c(C).$$

V is the set of coalitions S, for which i has decided, and S' and S" constituted that coalition.

For this concept the values of the three measures of difference (see Ståhl, p. 604) are:

1) The average sum of absolute difference 7,22
2) The average sum of squared differences 23,10
3) The average sum of the relative squared differences 1,67

4. Discussion

The concept presented here satisfied the "full cost" principle, the "individual rationality" principle and the "group rationality" principle, and thus produces allocations within "the core". The concept presented here does not guarantee that every party will become a member of a coalition, or even that only one coalition will be formed.

A coalition, S, is only formed when for two parties, each party being single or a coalition, S' and S":

$$(1) \qquad c(S) = c(S' \vee S'') < c(S') + c(S'')$$
$$\text{where } S', S'' \subset \{1,\ldots,n\} \text{ and } S' \wedge S'' = \{\emptyset\}.$$

However, condition (1) is not sufficient for the coalition $S' \vee S''$ to be formed. A necessary second condition is:

$$(2) \qquad \left\{\frac{c(S' \vee S'')}{c(S') + c(S'')} < \frac{c(S' \vee T')}{c(S') + c(T')}\right\} \wedge \left\{\frac{c(S' \vee S'')}{c(S') + c(S'')} < \frac{c(S'' \vee T'')}{c(S') + c(T'')}\right\}$$

for all alternative coalitions T' and T", where

$$S', S'', T', T'' \subset \{1,\ldots,n\} \text{ and } S' \wedge T' = \{\emptyset\},\ S'' \wedge T'' = \{\emptyset\}.$$

Conditions (1) and (2) are sufficient, provided that S' and S" are coalitions formed under these conditions, or singles.
The two parties, S' and S", of the coalition, S, are charged:

(3) $$c(S') \cdot \frac{c(S' \vee S'')}{c(S') + c(S'')} \text{ resp. } c(S'') \cdot \frac{c(S' \vee S'')}{c(S') + c(S'')}.$$

So the "full cost principle" is satisfied, since singles bear their go alone costs, $c(i)$.

From (1) it follows that

(4) $$\frac{c(S' \vee S'')}{c(S') + c(S'')} < 1.$$

Since for singles $x_i = c(i)$ and, according to (3) and (4), for coalition members (5): $x_i < c(i)$, the "individual rationality principle" is satisfied.

To prove that also the "group rationality principle" is satisfied, three cases will be distinguished. For each coalition S, a group of members, $S' \subset S$,

a) either formed a (smaller) coalition before forming S,

b) or did not form a (smaller) coalition before forming S, since for all the members of S' it was not 'individually rational' to do so.

Before proving that also the "group rationality principle" is satisfied, it is recalled that the concept presented here, does not guarantee the forming of a grand coalition. So we will prove that the "group rationality principle" is satisfied for each final coalition, S.

For each coalition, S, a group of members, $S' \subset S$, either formed a (smaller) coalition before forming S, or did not form a coalition before.

When they formed a coalition before, we have according to (3) and (4):

$$x_i = x(i \in S) < x(i \in S') \text{ for all } i \in S'.$$

So, in this case the "group rationality principle" is satisfied since the payments Σx_i are less than the costs of coalition S' on its own.

When they did not form a coalition before, two cases are distinguished:

a) Condition (1) was not satisfied, which implies $x(i \in S') > c(i)$ for all $i \in S'$, and since $x_i = x(i \in S) < c(i)$ for all $i \in S$, we have $x_i = x(i \in S) < x(i \in S')$ for all $i \in S'$. So, also in this case the "group rationality principle" is satisfied.

b) Condition (1) was satisfied, but condition (2) was not satisfied, for at least one member of S', who first joined a better alternative S", before joining S.

This member was single or joined S" as a member of coalition T. Denoting the costs of each of these two situations with c(T), we have:

$$c(T) \cdot \frac{c(S'')}{c(T) + c(S''-T)} < c(T) \cdot \frac{c(S')}{c(T) + c(S'-T)}$$

However, since T finaly joined S, we also have:

$$\frac{c(S)}{c(S'') + c(S-S'')} < 1.$$

So, the payments of this member T are clearly less than its share in the costs of the coalition S'.

What about the payments of S'-T? Some of these parties, S"-T, joined S" with T and are in the same situation as T. The rest, S'-S" is a single or formed a coalition like S", before joining S.

If S'-S" is a coalition it is in the same situation as S". Since also the single finally joined S, its payment is less than its go alone costs. And since its share in the costs of S' would also have been less than its go alone costs, the payments Σx_i of the members of S' are less than the costs of S' on its own. This concludes the proof that the "group rationality principle" is satisfied.

In his paper Ståhl discusses the choice among the three methods**) that produce core solutions: Nucleolus, Weak Nucleolus and Proportional

**) Another choice would be by the demand functions, in case customers are unwilling to pay any amount for fixed quantity of computer time. See Thijs ten Raa, "Supportability and Anonymous Equity", Journal of Economic Theory, September 1983.

Nucleolus. Since the Nucleolus violates the "Monotonicity principle", Ståhl rejects this method, and since the Weak Nucleolus violates the "Causality principle" he favored the Proportional Nucleolus.
The concept presented here satisfies both additional principles as well.

Since $x_i = c(i).\ c(S)/\sum_{i \in S} c(i)$, x_i varies proportionally with c(S) which proves that the "Monotonicity principle" is satisfied.
When a party never contributes to any cost savings, that party will remain single, which satisfies the "Causality principle".

In order to calculate the cost allocation according to this concept the costs of each possible coalition must be available. When these data are not available the procedure must be adapted to the available data. At least the "go alone" costs and the costs of the grand coalition must be known. In that case every party receives the same percentage reduction. In the example of Ståhl the allocation, only knowing these few data, becomes:

A	16,94
H	13,18
K	8,42
L	12,26
M	16,06
T	16,96
	83,82

A comparison of this allocation with the one based on all data, demonstrates that the data hold information about the contribution to costs savings by the various parties and coalitions: Both H and L but also K contribute substantially to the costs savings; the residual savings by A, M and T are relatively small. The concept of stepwise reduction takes that into account when this information is available.
The Nucleolus and The Weak Nucleolus do not take this information into account.

Furthermore the concept presented here also explains the formation of coalitions. This concept is in line with the experience that "in many games, a two or three-party coalition was first formed and then a five-party coalition, before the forming of the grand coalition"; Stn hl (1982, p. 605).
The average difference measures for the sixteen games gave values close to those for the Swedish game: Ståhl (1982, table 5).

5. Acknowledgement

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